OBJECTIVE MATHEMATICS

Volume 1

Descriptive Test Series

Prof. M. L. Khanna
Bhushan Muley

CHAPTER-2 : LOGARITHM

Unit Test-1

1. If $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y}$ and

$a^{y^2+yz+z^2} b^{z^2+zx+x^2} c^{x^2+xy+y^2} = \ldots$

2. The least value of the expression $2\log_{10} x - \log_x 0.01$ for $x > 1$ is ....

3. The value of a for which the equation

$$\left(\sqrt{a+\sqrt{a^2-1}}\right)^x + \left(\sqrt{a-\sqrt{a^2-1}}\right)^x = 2a$$

has only two solutions is ...

4. If $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$ where, $t = x^2 - 2|x|$ then the value of $x = \ldots$

5. If $a^x = b$, $b^y = c$, $c^z = a$,

$x = \log_b a^{k_1}$, $y = \log_c b^{k_2}$, $z = \log_a c^{k_3}$, find $k_1k_2k_3$.

6. Solve for x $\log_2 x \cdot \log_3 x \cdot \log_5 x = \log_2 x \cdot \log_3 x + \log_3 x \log_5 x + \log_5 x \log_2 x$

7. If $\log_2(a + b) + \log_2(c + d) \geq 4$, then the minimum value of $a + b + c + d$ is ...

Hints and Solutions

1. $\log P = \Sigma k(y - z)(y^2 + yz + z^2)$

$= \Sigma k(y^3 - z^3) = 0 \quad \therefore P = 1$

2. $-\log_x 10^{-2} = 2\log_x 10 = \frac{2}{\log_{10} x}$

L.H.S. $= 2\left(t + \frac{1}{t}\right) \geq 2 \cdot 2 = 4$ as A.M. ≥ G.M.

where, $t = \log_{10} x > 0$ as $x > 1$.

3. The two expressions are reciprocals of each other

If $\left[\sqrt{a+\sqrt{(a^2-1)}}\right]^x = t$

then $t + \frac{1}{t} = 2a$ or $t - 2at + 1 = 0$

$t = a\sqrt{a^2-1}, a - \sqrt{a^2-1}$

$\left(\sqrt{a+\sqrt{a^2-1}}\right)^x = \left(\sqrt{a+\sqrt{a^2-1}}\right)^2 \qquad \therefore x = 2$

or $= a - \sqrt{a^2-1} = \frac{1}{\left(\sqrt{a+\sqrt{a^2-1}}\right)^2}$

$\therefore \quad x = -2$

Provided $a^2 - 1$ is +ive *i.e.* $a > 1$.

4. $x^2 = |x|^2$

$y + \frac{1}{y} = 30$, where $y = (15 + 4\sqrt{14})^t$

or $\quad y^2 - 30y + 1 = 0$

$\therefore \quad y = 15 + 4\sqrt{14}, 15 - 4\sqrt{14}$

Now, $15 - 4\sqrt{14} = \frac{225-224}{15+4\sqrt{14}} = (15+4\sqrt{14})^{-1}$

$\therefore \quad t = 1, -1$

5. Since, $c^z = a$

$\Rightarrow \quad (b^y)^z = a$

$\Rightarrow \quad b^{yz} = a$

$\Rightarrow \quad (a^x)^{yz} = a \qquad \Rightarrow \quad a^{xyz} = a$

$\therefore \quad xyz = 1$

Also, $x = \log_a b$, $y = \log_a b$, $y = \log_b c$, $z = \log_c a$

$\Rightarrow$

$\log_b a^{k_1} = \log_a b, \log_c b^{k_2} = \log_b c, \log_a c^{k_3} = \log_c a$

$\Rightarrow \quad k_1 = \frac{\log_a b}{\log_b a}, k_2 = \frac{\log_b c}{\log_c b}, k_3 = \frac{\log_c a}{\log_a c}$

$k_1 = (\log_a b)^2, k_2 = (\log_b c)^2, k_3 = (\log_c a)^2$

$\Rightarrow \quad k_1 = x^2, k_2 = y^2, k_3 = z^2$

$\Rightarrow \quad k_1k_2k_3 = (xyz)^2$

$= (1)^2 = 1$

6. Let $\log_2 x = A$, $\log_3 x = B$, $\log_5 x = C$

$\therefore$ *Given* that, $ABC = AB + BC + CA$

$$\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = 1, \; ABC = 0$$

$A = 0$ or $B = 0$ or $C = 0$

$\therefore \quad x = 1$

$$\log_x 2 + \log_x 3 + \log_x 5 = 1$$

$$\log_x 30 = 1$$

$$x = 30$$

7. Given relation $\Rightarrow (a + b)(c + d) \geq 2^4 = 16$

Now, $\dfrac{(a+b)+(c+d)}{2} \geq [(a+b)(c+d)]^{1/2}$

$= 16^{1/2} = 4$

$\therefore \; (a + b)(c + d) \geq 2 \cdot 4$, *i.e.*, 8